A-Level Mathematics: Bridging Unit

The jump from GCSE to A-Level Mathematics can often be difficult and overwhelming. However, many units of work review and build upon topics already covered at GCSE. Using this document, you can review these areas so you begin your course with secure foundations for us to build on.

How to use this document:

1. This is divided into 10 key topics which are instrumental to you achieving success in A-Level Mathematics. For each of the topics, we have listed some of the topics that use these core foundations.

2. You complete a diagnostic test. You then self-mark this using the mark scheme provided.

3. You then reflect on how much work you need to do to in this topic to prepare for A-Level. Hegarty Maths clips will be provided. If you do not have access to Hegarty Maths, follow the link where Mr Hegarty has begun to provide YouTube tutorials on many of the areas covered in this document. https://www.youtube.com/user/HEGARTYMATHS. Alternatively, use the links on each page for tutorials from other sources.

4. When you think you have done enough preparation and revision of a particular topic there is a follow-up test. Again, there is a mark scheme for you to check your work.

5. If there are areas you still struggle with, make a note of these and the sort of questions you have and be prepared to share these with your A-Level teacher in the new academic year.

To summarise….

Diagnose

Revise

Test

Topic 1: Quadratic Equations

Quadratic Equations will link into many topics covered at A-Level:

- **The discriminant**
- **Modelling with quadratic equations**
- **Quadratic simultaneous equations**
- **Factor theorem and algebraic division**
- **Solving trigonometric equations**
- **Finding stationary points of curves**
- **Solving exponential functions**

Topic 1: Quadratic Equations

Step 1: Diagnostic Test

Question 1

Solve $x^2 + 6x + 8 = 0$ (2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

(1)

Total / 10

Topic 1: Quadratic Equations

Step 2: Revision

Topics

- Factorising quadratics
- Solving quadratic equations through factorisation
- Completing the square and solving quadratics
- Quadratic Formula

<u>Video Tutorials</u>

Factorising Quadratics
https://youtu.be/FRRkLZt31S0

Solving Quadratics by Factorising
https://youtu.be/m-qyV6C56ec

Factorising Harder Quadratics
https://youtu.be/v2sPNXdabI0

Quadratic Formula
https://youtu.be/hoy6dE0plLw

Completing the Square
https://youtu.be/9_2DYOfHxyk

Topic 1: Quadratic Equations

Step 3: Test Yourself

Question 1

Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2

(4)

Question 2

(i) Write $4x^2 - 24x + 27$ in the form of $a(x - b)^2 + c$

(4)

(ii) State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$.

(2)

Total / 10

Topic 2: Changing the Subject

Changing the subject will link into many topics covered at A-Level:

- **Solving simultaneous equations by substitution**
- **Equations of straight lines**
- **Parametric equations**
- **Iteration**

Topic 2: Changing the subject

Step 1: Diagnostic Test

Question 1

Make v the subject of the formula $E = \frac{1}{2}mv^2$

(3)

Question 2

Make r the subject of the formula $V = \frac{4}{3}\pi r^2$

(3)

Question 3

Make c the subject of the formula $P = \frac{c}{c+4}$

(4)

Total / 10

Topic 2: Changing the subject

Step 2: Revision

Topics

One and two steps → X's with negative coefficients → X's on the denominator → Powers and roots → X's on both sides → Problems also involving and substitution

Video Tutorials

Changing the Subject of a Formula

https://youtu.be/uLDLmsok-38

https://youtu.be/ldfTxszHGtg

Rearranging Harder Formulae

https://youtu.be/tQb9cF4xVeA

Topic 2: Changing the subject

Step 3: Test Yourself

Question 1

Make t the subject of the formula $s = \frac{1}{2}at^2$

(3)

Question 2

Make x the subject of $3x - 5y = y - mx$

(3)

Question 3

Make x the subject of the equation $y = \frac{x+3}{x-2}$

(4)

Total / 10

Topic 3: Simultaneous Equations

Simultaneous Equations will link into many topics covered at A-Level:

- **Linear simultaneous equations**
- **Non-linear simultaneous equations**
- **Simultaneous equations on graphs**
- **Problem solving where you are given two constraints**
- **Trigonometry - Rsin(x + a)**

Topic 3: Simultaneous Equations

Step 1: Diagnostic Test

Question 1

Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$

(3)

Question 2

Find the coordinates of the point of intersection of the lines $5x + 2y = 20$ and $y = 5 - x$

(3)

Question 3

Solve the simultaneous equations

$$x^2 + y^2 = 5$$

$$y = 3x + 1$$

(4)

Total / 10

Topic 3: Simultaneous Equations

Step 2: Revision

Topics

- Linear simultaneous equations (Elimination)
- Linear simultaneous equations (Substitution)
- Graphing simultaneous equations
- Quadratic simultaneous equations
- Graphing linear and non-linear simultaneous equations

Video Tutorials

Solving Simultaneous Equations

https://youtu.be/FcYeDDlgNMs

https://youtu.be/IYcvS7krDQc

Solving Simultaneous Equations Graphically

https://youtu.be/DBJ87TADmI0

Quadratic Simultaneous Equations

https://youtu.be/KSS0nLu5PnE

https://youtu.be/WxMuv8VdYzk

Topic 3: Simultaneous Equations

Step 3: Test Yourself

Question 1

Find the coordinates of the point of intersection of the lines $x + 2y = 5$ and $y = 5x - 1$

(3)

Question 2

The lines $y = 5x - a$ and $y = 2x + 18$ meet at the point $(7, b)$.

Find the values of a and b.

(3)

Question 3

A line and a curve has the following equations :

$$3x + 2y = 7 \qquad y = x^2 - 2x + 3$$

Find the coordinates of the points of intersection of the line and the curve by solving these simultaneous equations algebraically

(4)

Total / 10

Topic 4: Surds

Surds will link into many topics covered at A-Level:

- Surds and rationalising denominators
- The discriminant
- Equations of circles
- Trigonometric values and graphs
- Magnitude of vectors

Topic 4: Surds

Step 1: Diagnostic Test

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

(3)

Total / 10

Topic 4: Surds

Step 2: Revision

Topics

- Multiplication and division of surds
- Simplifying surds
- Expanding brackets involving surds
- Rationalising the denominator

Video Tutorials

Introduction to Surds
https://youtu.be/2_LkSaP2Vv8

Expanding and Simplifying Surds
https://youtu.be/Bqx5gYfsEGI

Rationalising the Denominator
https://youtu.be/t9iFBDHc-nA

Topic 4: Surds

Step 3: Test Yourself

Question 1

(i) Simplify $\sqrt{24} + \sqrt{6}$

(2)

(ii) Express $\frac{36}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

(3)

Question 2

(i) Simplify $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

(2)

(ii) Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

Total / 10

Topic 5: Indices

Indices will link into many topics covered at A-Level:

- **Index laws**
- **Quadratic equations involving indices**
- **Cubic and quartic graphs**
- **Dividing polynomials**
- **Binomial expansion**
- **Manipulation of expressions in order to differentiate and integrate**

Topic 5: Indices

Step 1: Diagnostic Test

Question 1

Simplify the following

(i) a^0 (1)

(ii) $a^6 \div a^{-2}$ (1)

(iii) $(9a^6b^2)^{-0.5}$ (3)

Question 2

(i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$ (2)

(ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ (3)

Total / 10

Topic 5: Indices

Step 2: Revision

Topics

- Powers of ‘0’ and ‘1’
- Powers of negative integers
- Multiplying indices
- Dividing indices
- Powers of powers
- Fractional indices

<u>Video Tutorials</u>

Introducing Laws of Indices

https://youtu.be/D_Q2R-I0Q1Y

Fractional and Negative Indices

https://youtu.be/omY_kulfPPg

Topic 5: Indices

Step 3: Test Yourself

Question 1

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$

(2)

(ii) $16^{\frac{3}{4}}$

(2)

Question 2

(i) Find a, given that $a^3 = 64x^{12}y^3$

(2)

(ii) $\left(\frac{1}{2}\right)^{-5}$

(2)

Question 3

Simplify $\frac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

(2)

Total / 10

Topic 6: Properties of lines

Straight lines will link into many topics covered at A-Level:

- **Modelling with straight line graphs**
- **Length and area problems with straight line graphs**
- **Intersection of straight lines and circles**
- **Tangent and chord properties**
- **Tangents and normals**
- **Modulus functions**
- **Composite, inverse and piecewise functions**

Topic 6: Properties of Lines

Step 1: Diagnostic Test

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

(3)

(ii) Find the length of AC

(2)

Question 2

Find, in the form y = mx + c, the equation of the line passing through A (3,7) and B (5,-1).

Show that the midpoint of AB lies on the line x + 2y = 10

(5)

Total / 10

Topic 6: Properties of Lines

Step 2: Revision

Topics

- Midpoints of line segments
- Gradients of line segments
- Straight line graphs (drawing and interpreting)
- Parallel & perpendicular lines

Video Tutorials:

Midpoint of a Line
https://youtu.be/Pt17ShlDFEo

Gradient of a Line
https://youtu.be/fn10eAtEX30

Equation of a Line
https://youtu.be/powhADI9SwA

Parallel and Perpendicular Lines
https://youtu.be/eFjXfm6vQY8

Topic 6: Properties of Lines

Step 3: Test Yourself

Question 1

The points A (-1,6), B (1,0) and C (13,4) are joined by straight lines. Prove that AB and BC are perpendicular.

(2)

Question 2

A and B are points with coordinates (-1,4) and (7,8) respectively. Find the coordinates of the midpoint, M, of AB.

(1)

Question 3

A line has gradient -4 and passes through the point (2,-6). Find the coordinates of its points of intersection with the axes.

(4)

Question 4

Find the equation of the line which is parallel to $y = 3x + 1$ and which passes through the point with coordinates (4,5).

(3)

Total / 10

Topic 7: Sketching Curves

Sketching Curves will link into many topics covered at A-Level:

- **The discriminant**
- **Representing quadratic inequalities and simultaneous equations on graphs**
- **Factor theorem**
- **Solving trigonometric equations**
- **Sketching gradient functions**
- **Sec(x), Cosec(X), Cot(X) and Arcsin(x), Arccos(x), Arctan(x)**
- **Parametric Equations**

Topic 7: Sketching curves

Step 1: Diagnostic Test

Question 1

In the cubic polynomial f(x), the coefficient of x^3 is 1. The roots of f(x) = 0 are -1, 2 and 5.

Sketch the graph of y = f(x)

(3)

Question 2

Sketch the graph of $y = 9 - x^2$

(3)

Question 3

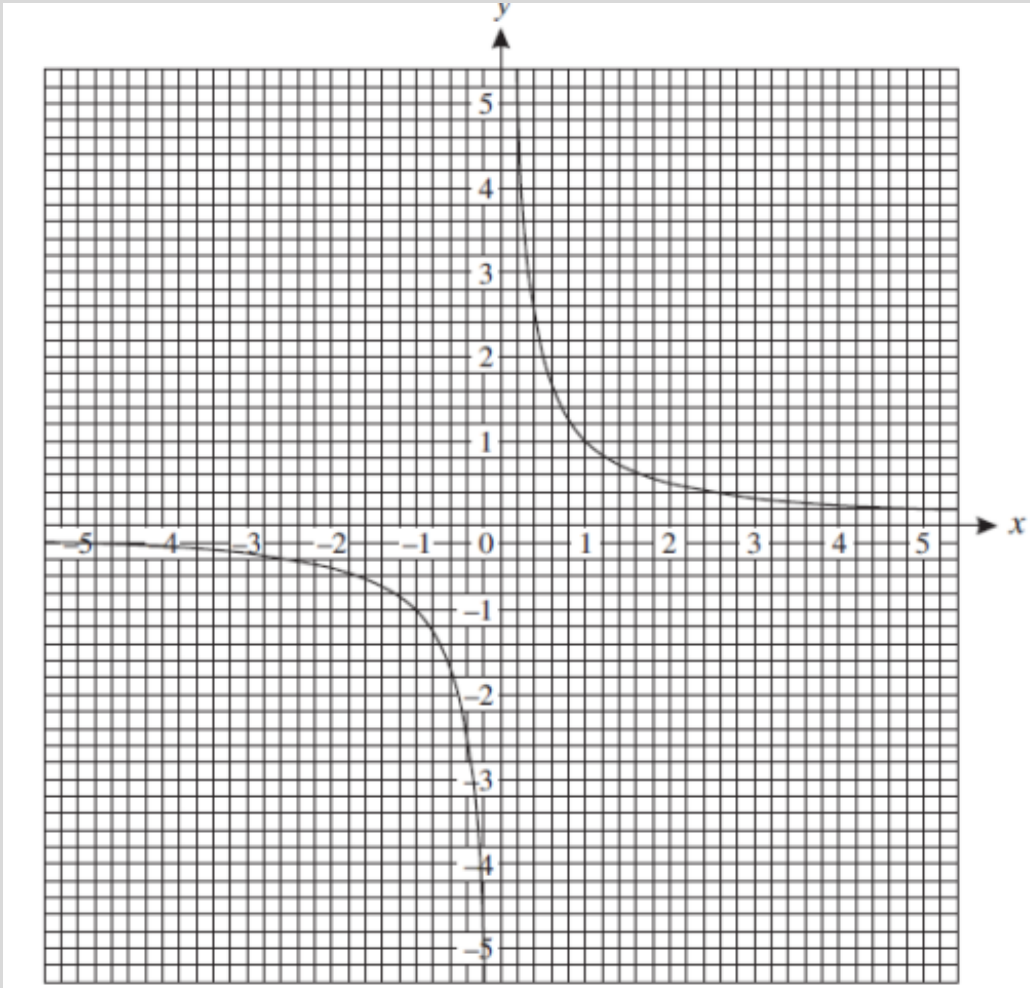
The graph below shows the graph of $y = \frac{1}{x}$

On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \leq x \leq 5$

Graph on next page…

Topic 7: Sketching curves

Step 1: Diagnostic Test



(4)

Total / 10

Topic 7: Sketching curves

Step 2: Revision

Topics

Quadratic graphs → Cubic graphs → Reciprocal graphs

Video Tutorials

Drawing Quadratic Graphs

https://youtu.be/rM_A8t0CV7A

Drawing Cubic and Reciprocal Graphs

https://youtu.be/P11WAJoBCVY

Topic 7: Sketching curves

Step 3: Test Yourself

Question 1

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$

Sketch the graph of $y = f(x)$

(3)

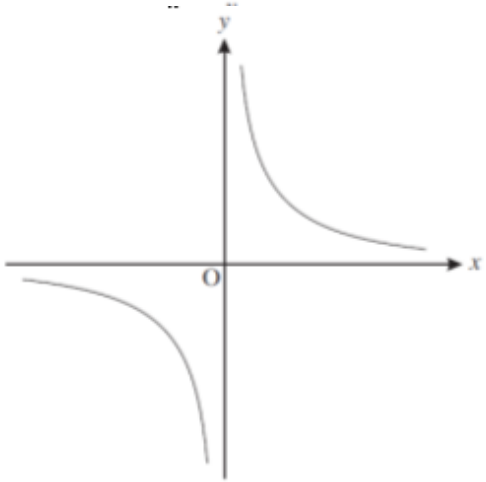
Question 2

Sketch the graph of $y = x(x - 3)^2$

(3)

Question 3

This diagram shows a sketch of the graph of $y = \frac{1}{x}$



Sketch the graph of $y = \frac{1}{x-2}$, showing clearly any points where it crosses the axes.

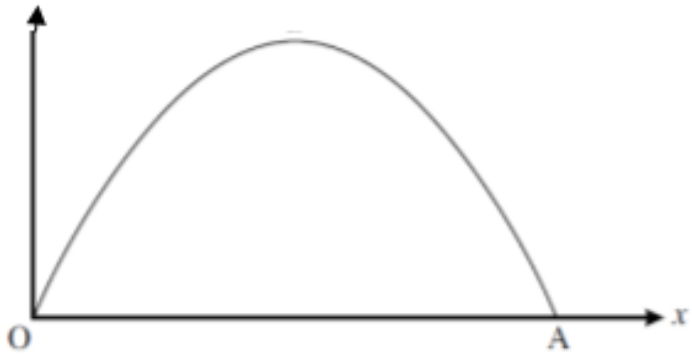
More on next page...

Topic 7: Sketching curves

Step 3: Test Yourself

Question 4

This curve has equation $y = \frac{1}{5}x\,(10 - x)$. State the value of x at the point A.



(1)

Total / 10

Topic 8: Transformations of Functions

Transformation of functions will link into many topics covered at A-Level:

- **Translating and transforming graphs**
- **Graphs of sine, cosine and tangent**
- **Solving trigonometric functions of the form sin(nx), cos(nx) and tan(nx)**
- **Exponential functions**

Topic 8: Transformation of Functions

Step 1: Diagnostic Test

Question 1

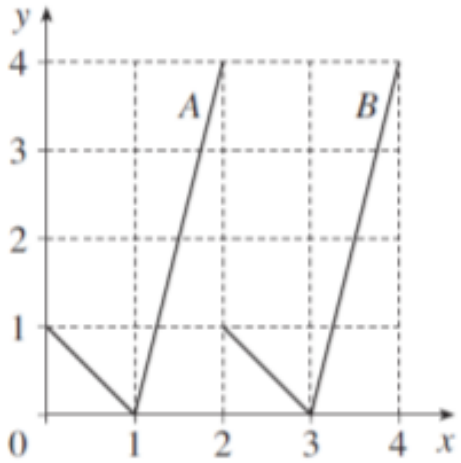
The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Write down an equation for the translated curve. You need not simplify your answer.

(2)

Question 2

This diagram shows graphs A and B.



(i) State the transformation which maps graph A onto graph B

(2)

More on next page...

Topic 8: Transformation of Functions

Step 1: Diagnostic Test

(ii) The equation of graph A is $y = f(x)$.

Which one of the following is the equation of graph B ?

$y = f(x) + 2$	$y = f(x) - 2$	$y = f(x+2)$	$y = f(x-2)$
$y = 2f(x)$	$y = f(x+3)$	$y = f(x-3)$	$y = 3f(x)$

(2)

Question 3

(i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

(2)

(ii) Sketch the graph of $y = x^2 - 4$

(2)

Total / 10

Topic 8: Transformation of Functions

Step 2: Revision

Topics

- f(x)±a and f(x±a)
- af(x) and f(ax)
- f(-x) and –f(x)
- Combining transformations

<u>Video Tutorials</u>

Transforming Graphs

https://youtu.be/F8YGp_j7YhM

https://youtu.be/8URwwX6PIQI

Topic 8: Transformation of Functions

Step 3: Test Yourself

Question 1

The graph of $y = x^2 - 8x + 25$ is translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$. State an equation for the resultant graph.

(1)

Question 2

$f(x) = x^3 - 5x + 2$

Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$

(4)

Question 3

You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$

Show that $f(x - 4) = 2x^3 - 17x^2 + 33x$

(3)

Question 4

You are given that $f(x) = (x + 1)(x - 2)(x - 4)$.

The graph of $y = f(x)$ is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

(2)

Total / 10

Topic 9: Trigonometric Ratios

Trigonometric Ratios will link into many topics covered at A-Level:

- **Cosine & Sine rule**
- **Area of triangles**
- **Trigonometric graphs**
- **Solving trigonometric equations**
- **Differentiating trigonometric functions**
- **Integrating trigonometric functions**

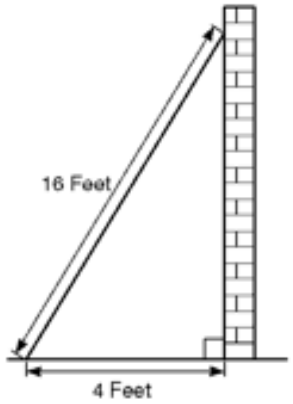
Topic 9: Trigonometric Ratios

Step 1: Diagnostic Test

Question 1

Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall.

The ladder is 16 feet long.



The foot of the ladder is 4 feet from the base of the wall.

(i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

(2)

(ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 significant figures

(2)

More on next page…

Topic 9: Trigonometric Ratios

Step 1: Diagnostic Test

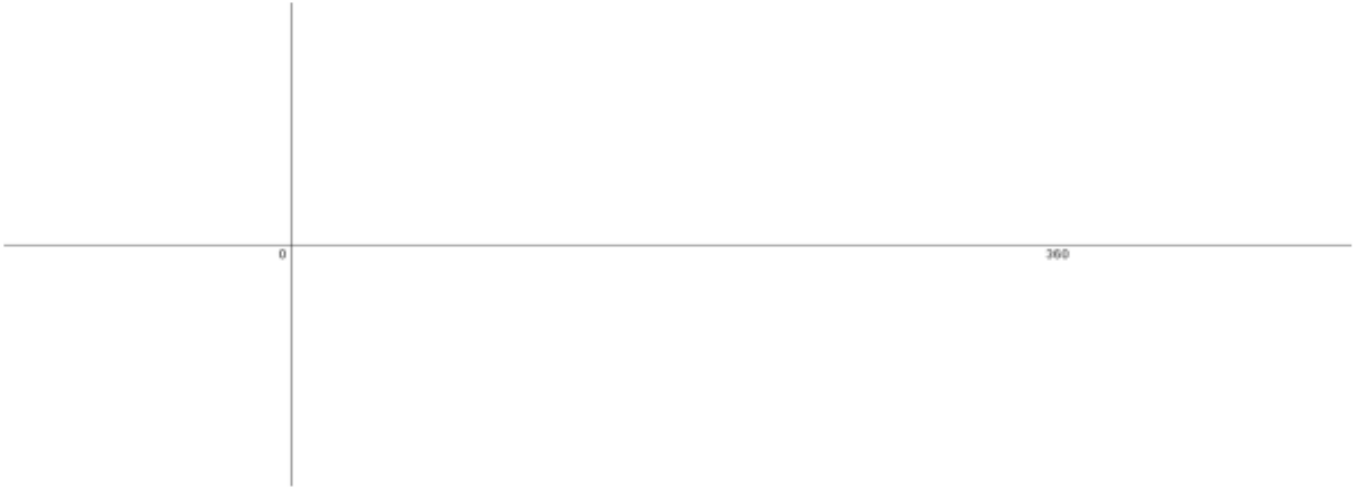
Question 2

Given that $\cos \Theta = \frac{1}{3}$ and Θ is acute, find the exact value of $\tan \Theta$

(3)

Question 3

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360°$



(3)

Total / 10

Topic 9: Trigonometric Ratios

Step 2: Revision

Topics

- Finding missing sides and angles
- Multi-step problems
- Angles of elevation and depression
- Trigonometric graphs
- Non-calculator trigonometry

Video Tutorials

Right-Angled Trigonometry

https://youtu.be/Ka7KyDTMuKA

https://youtu.be/ag7B4PEAmKw

Exact Trigonometric Values

https://youtu.be/3KYomDEIQgo

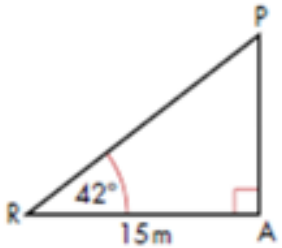
https://youtu.be/9uAbKE125_E

Topic 9: Trigonometric Ratios

Step 3: Test Yourself

Question 1

AP is a telephone pole. The angle of elevation of the top of the pole from the point R on the ground is 42°as seen in the diagram.



Calculate the height of the pole. Give your answer to 3 significant figures.

(3)

Question 2

Given that $\sin \Theta = \frac{\sqrt{3}}{4}$, find in surd form the possible values of $\cos \Theta$.

(3)

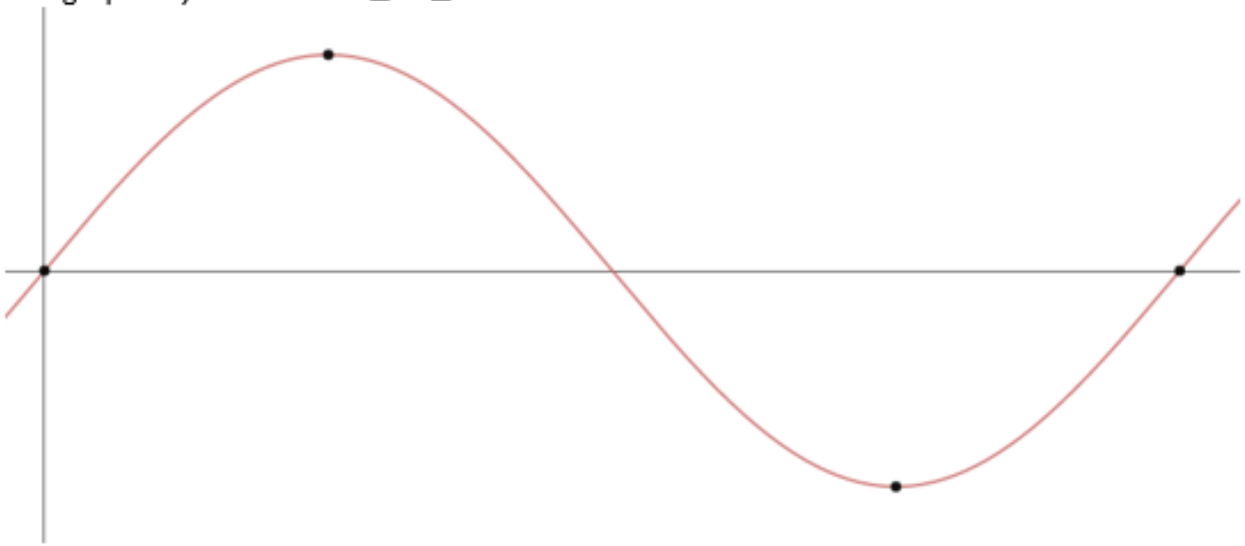
More on next page...

Topic 9: Trigonometric Ratios

Step 3: Test Yourself

Question 3

The graph of $y = \sin x$ for $0 \leq x \leq 360°$ is shown below.



What are the coordinates of the 4 points labelled on the graph?

(……, ……)
(……, ……)
(……, ……)
(……, ……)

(4)

Total / 10

Topic 10: Sine & Cosine Rule

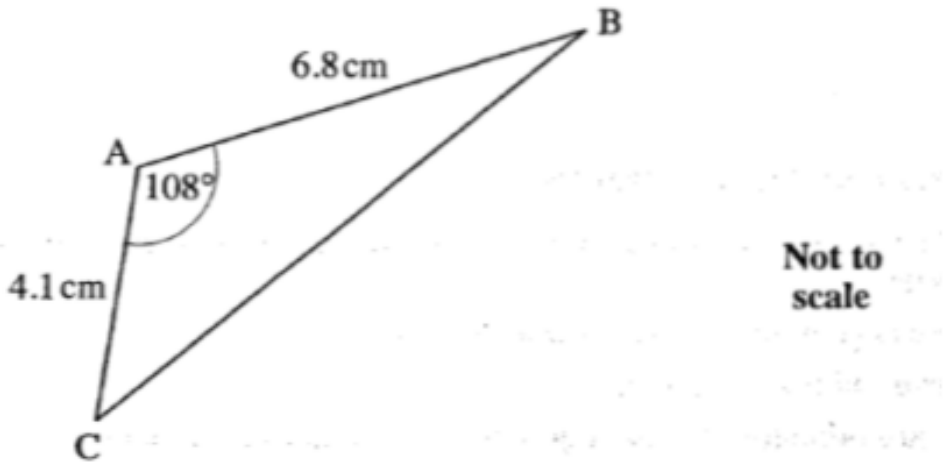
Sine & Cosine Rule will link into many topics covered at A-Level:

- **Area of triangles**
- **Vectors**

Topic 10: Sine & Cosine Rule

Step 1: Diagnostic Test

Question 1



For triangle ABC, calculate

(i) the length of BC

(3)

(ii) the area of triangle ABC

(3)

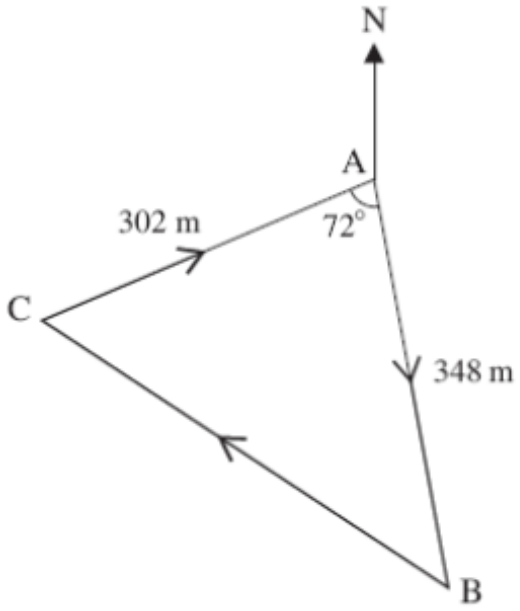
More on next page…

Topic 10: Sine & Cosine Rule

Step 1: Diagnostic Test

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Not to scale

Calculate the total length of the course for this race.

(4)

Total / 10

Topic 10: Sine & Cosine Rule

Step 2: Revision

Topics

Area of a triangle → Sine rule → Cosine rule → Bearings → Multi-step problems

Video Tutorials:

Area of a Triangle Using Sine
https://youtu.be/xWIsDBPI0Gg

The Sine Rule
https://youtu.be/4vQqDLu86qg

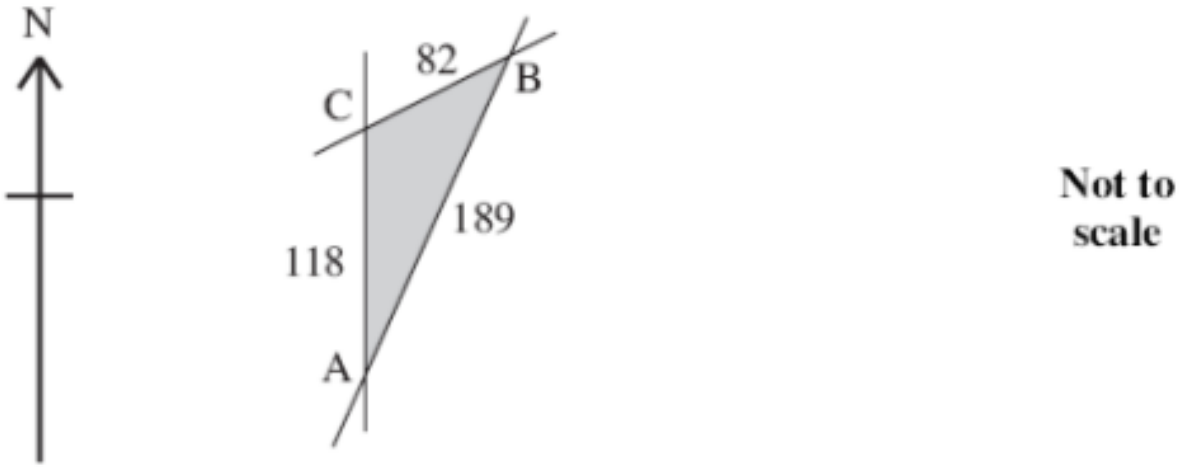
The Cosine Rule
https://youtu.be/ud8fUf7Wlzc
https://youtu.be/U2sNPD7KOyw

Topic 10: Sine & Cosine Rule

Step 3: Test Yourself

Question 1

This diagram shows a village green which is bordered by 3 straight roads AB, BC and AC. The road AC runs due North and the measurements are shown in metres.



Not to scale

(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°

(4)

(ii) Calculate the area of the village green.

(2)

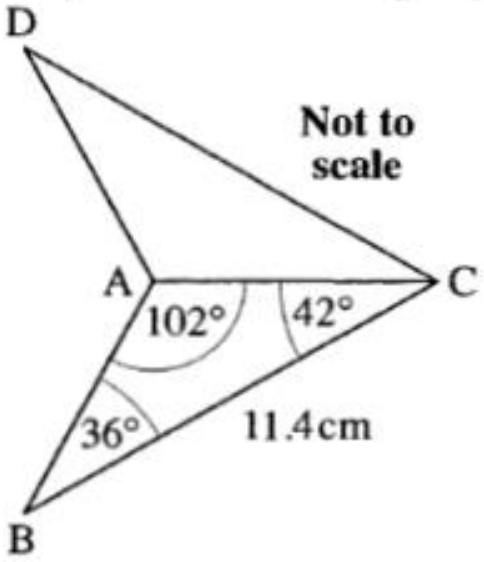
Topic 10: Sine & Cosine Rule

Step 3: Test Yourself

Question 2

This diagram shows a logo ABCD. It is symmetrical about AC.

Find the length of AB and hence find the area of the logo



(4)

Total / 10